CENTRIFUGAL EFFECTS WHEN A PLASMA IS ROTATED IN A FLAT HOMOPOLAR DEVICE

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We have investigated the effect of secondary overflows on the basic azimuthal motion of a viscous plasma.

In an experimental study of a weakly ionized gas-discharge plasma in a homopolar device [1] we observed certain phenomena that were unexpected from the standpoint of conventional hydrodynamics: the shift in polarity led to a pronounced difference in the distribution of azimuthal velocity in the plane of symmetry of the installation.

In the general case, when a plasma is rotated in a homopolar device, the motion is three-dimensional, i.e., in the plane normal to the basic motion we find secondary overflows which may significantly affect the entire flow pattern. These overflows arise in the motion of a viscous continuous medium in any curvilinear channel and are a result of the gradient of centrifugal forces in the direction that coincides with the axis of symmetry of the installation. Moreover, the nature of these overflows may be affected by the ion wind [1, 2] when we have the motion of a plasma with magnetized electrons.

However, for simplicity, let us initially examine the motion of a nonmagnetized plasma and we will try to isolate the influence of the centrifugal effect exclusively as it pertains to the main flow in some special case, without consideration of other factors, equally important in the general case.

Let us assume that the following conditions are satisfied. The flow regime is laminar and the temperature is constant throughout the entire volume, thus also making it possible to hold that the viscosity is constant; the plasma is incompressible. In the usual regimes the following conditions are valid: pressure $p \approx 1$ torr; current $I \sim 1$ A; magnetic field strength ~ 2500 Oe. The temperature in this case is of the order of 700°K, the Mach number $M \lesssim 0.2$, and the Reynolds numbers are small $(10 \leq \mathrm{Re} \leq 300)$ [1, 3].

Since the plasma is weakly ionized, we can neglect the induced electric current (this also follows from an examination of the equations of momentum balance for the electron and ion component of the plasma) and we can treat the current-density distribution as specified; in addition, Ohm's law can be presented in the form $j = \sigma E$.

Let us examine the rotation of a plasma in an axial electric field and in a radial magnetic field (in a so-called inverted homopolar device) under the action of the Lorentz force $j_z B_r$ (here j_z is the density of the axial current and B_r is the radial magnetic field). Considering the above and bearing in mind the axial symmetry, we write [4] the equations of motion

$$u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=\nu\left(\frac{\partial^2 v}{\partial z^2}+\frac{\partial^2 v}{\partial r^2}+\frac{\partial v}{r\partial r}-\frac{v}{r^2}\right)+\frac{j_z B_r}{\rho}, \tag{1}$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=-\frac{\partial p}{\rho\partial r}+\nu\left(\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 u}{\partial r^2}+\frac{\partial u}{r\partial r}-\frac{u}{r^2}\right), \tag{2}$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\rho\partial z}+\nu\left(\frac{\partial^2 w}{\partial z^2}+\frac{\partial^2 w}{\partial r^2}+\frac{\partial w}{r\partial r}\right), \tag{3}$$

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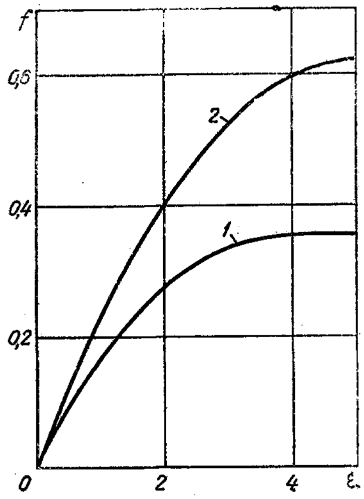

Fig. 1

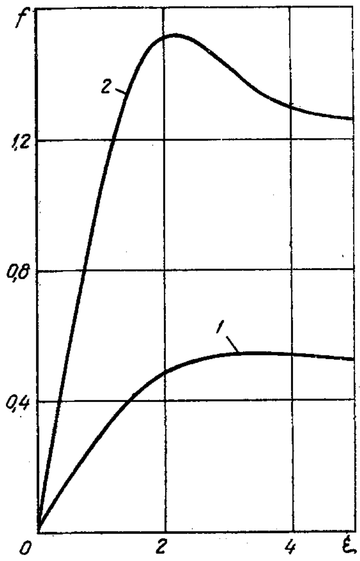

Fig. 2

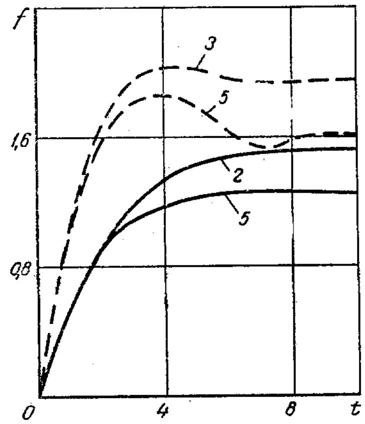

Fig. 3

Fig. 1. Profiles of the dimensionless velocity when $\alpha = 0.05$: 1) with consideration of secondary overflows; 2) one-dimensional motion.

Fig. 2. Distribution of dimensionless velocity: 1) when $\alpha = 0.1$; 2) 0.5.

Fig. 3. The process of establishing the function (the solid curve denotes $\alpha = 0.5$ for $\xi = 2$ and 5; the dashed curve denotes $\alpha = 1.0$ for $\xi = 3$ and 5).

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0. \tag{4}$$

We assume in Eq. (1) that

$$j_z = j_1 \left(\frac{r}{r_1}\right)^2, \tag{5}$$

where j_1 is the current density near the surface of the inside dielectric cylinder. This distribution is possible, for example, with segmented electrodes [5].

The system of equations (1)-(5) must be solved in conjunction with the Maxwell equations:

$$\frac{\partial \mathbf{B}}{\partial t} = -\operatorname{rot} \mathbf{E} = 0, \quad \operatorname{div} \mathbf{j} = 0, \quad \mathbf{j} = \operatorname{rot} \mathbf{H}, \quad \operatorname{div} \mathbf{B} = 0. \tag{6}$$

According to the last expression, $B_r = B_1(r_1/r)$. We will demonstrate that relationship (5) is compatible with the Maxwell equations, with an accuracy to the second order of smallness. Indeed, in the light of the linearity of these equations, for a medium with constant physical properties we can represent the density of the axial current in the following form:

$$j_z = j_1 \left(\frac{r}{r_1}\right)^2 + \Phi(z, r). \tag{7}$$

Using (7) in conjunction with the first two equations in (6) and Ohm's law, we derive the new equation

$$\frac{d^2\Phi}{dz^2} + \frac{4j_1}{r_1^2} = 0, \tag{8}$$

whose solution is given by the expression

$$\Phi = 2j_1 \left(\frac{h}{2r_1}\right)^2 (1 - \xi^2). \tag{9}$$

Here $z = (h/2)\xi$ and we use the boundary conditions

$$\left(\frac{d\Phi}{d\xi}\right)_0 = 0, \quad \Phi = 0 \text{ when } \xi = \pm 1.$$

Since it is the motion of a plasma in a flat homopolar device that we are investigating, and since the condition $((h/2)/r_1)^2 \ll 1$ is satisfied for this case, with an accuracy to the second order of smallness in Eq. (7) we can neglect the function Φ defined by relationship (9).

Let us now examine Eqs. (1)-(4). For convenience, we turn to the dimensionless parameters

$$\bar{v} = \frac{v}{\Omega l}, \quad \bar{u} = \frac{u}{\Omega l}, \quad \bar{w} = \frac{w}{\Omega l}, \quad \zeta = \frac{r}{l}, \quad \xi = \frac{z}{l},$$

where Ω and l are certain characteristic values of the angular velocity and length.

Let us write the expressions for the dimensionless stream function

$$\zeta\bar{u} = \frac{\partial\psi}{\partial\xi}, \quad \zeta\bar{w} = -\frac{\partial\psi}{\partial\zeta}. \tag{10}$$

Having introduced all of these dimensionless parameters into system (1)-(4), having eliminated the pressure in advance, we obtain the following equations:

$$\begin{aligned} f'' &= 2f\omega' - 2f'\omega - \alpha, \\ \omega^{\mathrm{IV}} &= -2\omega\omega''' - 2ff'. \end{aligned} \tag{11}$$

This system is valid for a fairly large distance between the dielectric cylinders in comparison with the gap between these when it is possible to neglect the effect of the viscosity on coaxial cylinders [6]. The functions f and ω are associated with the dimensionless velocity $\bar{v}$ and the stream function ψ by the relationships

$$\bar{v} = \zeta f(\xi), \quad \psi = \zeta^2\omega(\xi). \tag{12}$$

For the characteristic angular velocity we have chosen $\Omega = \nu/l^2$; $\alpha = j_1 B_1/\Omega^2 \rho r_1$ is the parameter of electromagnetic interaction.

The boundary conditions will be defined by an absence of slip at the end faces of the homopolar device. Then, directing the ξ-axis from the lower end face of the installation toward the upper end face, and bearing in mind the symmetry of the plasma stream relative to the equatorial plane, we will find that

$$\text{when } \xi = 0 \quad f = \omega = \omega' = 0, \text{ when } \xi = \xi_1 \quad f' = \omega = \omega'' = 0. \tag{13}$$

To solve the system of equations (11) with the boundary conditions (13) we employed approximate methods of calculation. The solutions for f and ω were presented in the form of finite series:

$$f = \sum_{n=0}^{k} C_n f_n, \quad \omega = \sum_{n=0}^{m} \varepsilon_n \psi_n, \tag{14}$$

in which f_n and ψ_n are certain basic functions which satisfy the boundary conditions. Consequently, solutions (14) satisfy these conditions automatically. We have chosen the following expressions as the basic functions:

$$\begin{aligned} f_n &= 5\,\frac{n+2}{n+1}\,\xi^{n+1} - \xi^{n+2}, \\ \psi_n &= \xi^{n+4} - 5\,\frac{2n+5}{n+2}\,\xi^{n+3} + 25\,\frac{n+3}{n+2}\,\xi^{n+2}. \end{aligned} \tag{15}$$

As the collocation points at which the differential equations (11) are satisfied exactly we have chosen the points 1, 2, 3, 4, and 5, i.e., ξ changes in the range $0 \leq \xi \leq 5$. Moreover, the following conditions are satisfied exactly:

$$f'''(5) = \omega^{\mathrm{IV}}(5) = 0.$$

To find the coefficients C_n and ε_n independent of ξ, we employed the settling method. This method is based on the Navier–Stokes equations for nonsteady motion, and these, after transformation, assume the form

$$\frac{\partial f}{\partial t}=\frac{\partial^2 f}{\partial \xi^2}+2\omega\frac{\partial f}{\partial \xi}-2f\frac{\partial \omega}{\partial \xi}+\alpha,$$
$$\frac{\partial^3 \omega}{\partial t\,\partial \xi^2}=\frac{\partial^4 \omega}{\partial \xi^4}+2\omega\frac{\partial^3 \omega}{\partial \xi^3}+2f\frac{\partial f}{\partial \xi} \qquad (16)$$

which change into Eq. (11) as $t\to\infty$.

An electronic digital computer was used to solve the problem and the following basic results were obtained.

The influence of the secondary overflows on the main flow become noticeable even when $\alpha \geq 0.01$. When $\alpha = 0.05$ (Fig. 1) the value of the dimensionless function f in the symmetry plane diminished by a factor of almost 2 relative to the value derived in the assumption that there are no secondary flows. When $\alpha = 0.1$ (Fig. 2) we find a qualitative change in the velocity profile: the maximum value of the function shifts from the symmetry plane in the direction of the end face. With an increase in α this trend is intensified. The maximum value of the velocity function, for example, when $\alpha = 0.5$, is smaller by a factor of approximately 5 than the quantity calculated in the one-dimensional approximation.

The total reduction in azimuthal velocity results from the partial transformation of the azimuthal momentum into the momentum of the secondary overflows, as well as from the losses resulting from the viscous friction in these overflows. The qualitative change can be explained by the fact that the radial velocity in the symmetry plane is directed outward, so that the segments of the medium moving from a zone at a smaller tangential velocity shift to the zone with a greater value for the tangential velocity, and we find a reduction in the azimuthal velocity. Approximately at the midpoint between the end face and the symmetry plane the radial velocity changes sign and the reverse phenomenon occurs, i.e., there is an increase in the azimuthal velocity in comparison to its reduction in the equatorial plane.

The development of a strong momentum for the secondary overflows changes the pattern of motion that prevails in the process of plasma acceleration. When $\alpha < 0.5$ the values of f in the interval 0-ξ_1 asymptotically tend to the limit. When $\alpha = 1$ it is apparently impossible for the secondary flows to develop a rapidly increasing gradient of centrifugal forces. The function f therefore attains values somewhat greater than for the steady state, and an oscillatory process develops, which, however, is rapidly attenuated. This is shown in Fig. 3 for values of the function f for $\xi = 3$ and 5. The oscillatory process is barely noticeable, even when $\alpha = 0.5$.

NOTATION

- j is the density vector of the electric current;
- σ is the electrical conductivity;
- **E** is the vector of the electric field strength;
- u is the radial velocity component;
- v is the azimuthal component;
- w is the axial component;
- ν is the kinematic viscosity;
- ρ is the density;
- p is the pressure;
- r is the radius;
- z is the coordinate along the instrument axis;
- **H** is the vector of the magnetic field intensity;
- **B** is the magnetic induction.

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